Potential Solution to the Vacuum Energy and Gauge Hierarchy Problems

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The Problems

Huge Vacuum Energy

Theoretical zero-point energy (ZPE) differs from observed energy density of universe by factor of 10^{120} (or 10^{60} by Lorentz invariant analysis)

Gauge Hierarchy Problem

Radiative corrections should drive
Higgs mass to Planck energy level

Possible Solutions

Supersymmetry (SUSY)

Possible Solutions

Attempts to Solve the Problems of the Standard Model

Many attemps going in different directions

- Extended gauge symmetries
- New mechanisms of symmetry breaking
- More fundamental fields
- Extra dimensions
- ...

Historical Digression

Circa 1998: Bangalore India golf course

Sitting on veranda sipping cool drinks and deducing Dirac equation solutions from scratch, while my wife golfed.

Found 8 solutions in all, not 4, as in standard model.

Half of solutions had $-E_p$ in spinor components

Soon realized similar thing for K-G and Maxwell equations

QFT Background

Brief History of Quantum Energy

Non-Relativistic Quantum Mechanics (NRQM)

Shroedinger equation solution $\psi_{\mathbf{p}} = A_{\mathbf{p}} e^{-i(Et - \mathbf{p}\bullet\mathbf{x})}$

State (particle)

Relativistic Quantum Mechanics (RQM)

Klein-Gordon solution $\phi_{\mathbf{p}} = \phi_{\mathbf{p}}^{+} + \phi_{\mathbf{p}}^{-} = A_{\mathbf{p}} e^{-i(Et - \mathbf{p}\bullet\mathbf{x})} + B_{\mathbf{p}} e^{+i(Et - \mathbf{p}\bullet\mathbf{x})}$

State (particle)

Quantum Field Theory (QFT)

Klein-Gordon solution $\phi_{\mathbf{k}} = a(\mathbf{k}) e^{-i(\omega_{\mathbf{k}} t - \mathbf{k}\bullet\mathbf{x})} + b^{\dagger}(\mathbf{k}) e^{i(\omega_{\mathbf{k}} t - \mathbf{k}\bullet\mathbf{x})}$

Quantum field

Non-Relativistic Quantum Mechanics (NRQM)

$$\text{Hamiltonian } H = i\frac{\partial}{\partial t} \qquad H\phi_{\mathbf{p}} = i\frac{\partial \phi_{\mathbf{p}}}{\partial t} = E_{\mathbf{p}}\phi$$

Energies positive

Relativistic Quantum Mechanics (RQM)

$$\text{Hamiltonian } H = i\frac{\partial}{\partial t} \qquad H\phi^{\pm}_{\mathbf{p}} = i\frac{\partial \phi^{\pm}_{\mathbf{p}}}{\partial t} = \pm E_{\mathbf{p}}\phi^{\pm}_{\mathbf{p}}$$

Energies positive and negative

Quantum Field Theory (QFT)

$$E_{\mathbf{k}} = \hbar\omega_{\mathbf{k}} \quad \mathbf{p} = \hbar\mathbf{k} \quad \hbar = 1$$

$$H = \int \mathcal{H}\, dV = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(N_a(\mathbf{k}) + \tfrac{1}{2} + N_b(\mathbf{k}) + \tfrac{1}{2} \right) \qquad H\left|\phi_{\mathbf{k}}\right\rangle = \omega_{\mathbf{k}}\left|\phi_{\mathbf{k}}\right\rangle$$

Energies positive

Non-Relativistic Quantum Mechanics (NRQM)

Vacuum energy = 0

Relativistic Quantum Mechanics (RQM)

Vacuum energy = 0

Quantum Field Theory (QFT)

$$H|0\rangle = \sum_{\mathbf{k}} \left(\tfrac{1}{2}\omega_{\mathbf{k}} + \tfrac{1}{2}\omega_{\mathbf{k}} \right) |0\rangle = \infty |0\rangle$$

Vacuum energy (ZPE) = ∞ or humongous

Unused Solutions to the Field Equations

QFT Supplemental Solutions

Take $\omega_{\mathbf{k}} t$ to $-\omega_{\mathbf{k}} t$ in traditional solutions

$$\underline{\phi} \propto \sum_{\mathbf{k}} \left\{ \underline{a}(\mathbf{k}) e^{i(\omega_{\mathbf{k}} t + \mathbf{k}\bullet\mathbf{x})} + \underline{b}^{\dagger}(\mathbf{k}) e^{-i(\omega_{\mathbf{k}} t + \mathbf{k}\bullet\mathbf{x})} \right\}$$

Also solves K-G equation

$$\left(\partial_{\mu}\partial^{\mu} + m^2\right)\underline{\phi} = \left(\partial_0\partial_0 - \partial_1\partial_1 - \partial_2\partial_2 - \partial_3\partial_3 + m^2\right)\underline{\phi} = 0$$

https://arxiv.org/abs/1802.03277

Crank the QFT Math

Vacuum Energy for Supplementals

$$\underline{H}\,|0\rangle = -\sum_{\mathbf{k}} \left(\tfrac{1}{2}\omega_{\mathbf{k}} + \tfrac{1}{2}\omega_{\mathbf{k}} \right) |0\rangle = -\infty |0\rangle$$

∞ negative vacuum energy

Real Particle Energy for Supplementals

$$\underline{H}\left|\underline{\phi}_{\mathbf{k}}\right\rangle = -\omega_{\mathbf{k}}\left|\underline{\phi}_{\mathbf{k}}\right\rangle$$

Negative energy

Not surprising as took $\omega_{\mathbf{k}} \rightarrow -\omega_{\mathbf{k}}$

3-momentum

Opposite direction of velocity

Velocity of traditional wave $e^{-i(\omega_{\mathbf{k}}t-kx)}=e^{-i\left(E_{\mathbf{p}}t-px\right)}\rightarrow E_{\mathbf{p}}t-px=\text{constant}$

$$v=\frac{dx}{dt}=\frac{E_{\mathbf{p}}}{p}$$ p and v in same direction

Velocity of supplemental wave $e^{-i(\omega_{\mathbf{k}}t+kx)}=e^{-i\left(E_{\mathbf{p}}t+px\right)}\rightarrow E_{\mathbf{p}}t+px=\text{constant}$

$$v=\frac{dx}{dt}=-\frac{E_{\mathbf{p}}}{p}$$ p and v in oppsite direction

Above is NRQM & RQM analysis. Same result, but more complicated analysis, in QFT.

Propagators

Opposite sign from traditional field propagators.

Direction in Time

Behave like SM particles backward in time.

Can interpret $\omega_{\mathbf{k}} t$ to $-\omega_{\mathbf{k}} t$ as t to $-t$

Negative energy forward in time or positive energy backward in time

Supplemental Scalars

As above.

Supplemental Fermions

Similar to scalars: Opposite sign on ZPE, propagators, etc.

Supplemental Vector Bosons

Similar to scalars: Opposite sign on ZPE, propagators, etc.

Conjectures

Supplemental particles

- *Uncoupled from SM fermions and gauge bosons*
- *Possibly coupled to Higgs*
- *Possibly coupled to gravity*
- *Only exist in our (forward time) universe as virtual particles*
  - *(Virtual particles in SM can have positive or negative energy and 3-momentum in any direction)*

Huge Vacuum Energy Resolved?

Vacuum Energy with Supplementals

$$H_{total}|0\rangle = (H + \underline{H})|0\rangle = (\infty - \infty)|0\rangle = 0$$

Total vacuum energy = 0

No Exotic New Math/Theory

Simply include solution forms that already exist in the basic mathematics. Nothing more. No fancy, convoluted theory.

QFT, as we know and love it, already contains zero vacuum energy. Overlooking valid solution forms to the field equations may be the source of the vacuum energy problem.

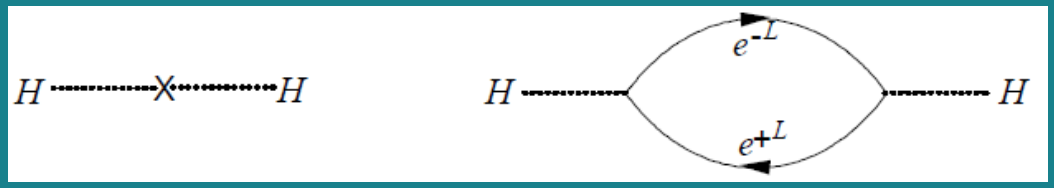
How About Gauge Hierarchy?

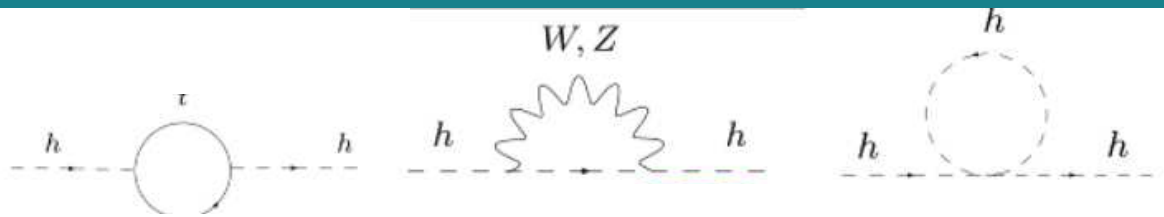
Brief History of Gauge Hierarchy

Radiative corrections to Higgs mass.

Lagrangian Higgs mass term = $\mu^2 HH$

Radiative correction term





$$m_H^2 = |2\mu|^2 - \Delta m^2_{corrects} = 125^2 = |2\mu|^2 - \frac{\Lambda^2}{6\pi^2}$$

Λ can be as large as the Planck scale

36127890984789307394520932878928933023 -
36127890984789307394520932878928917398 =
$= m^2_H = 125^2$

Is this natural ?

pinch of salt

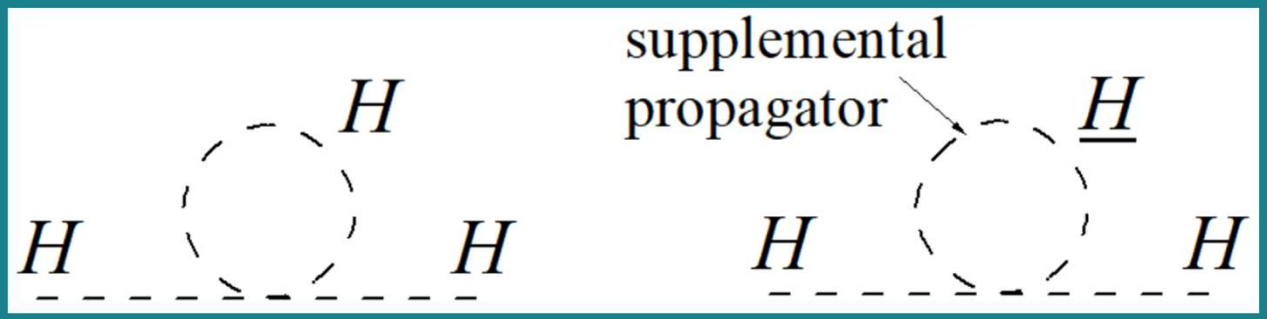
=

9.999999… billion tons of salt

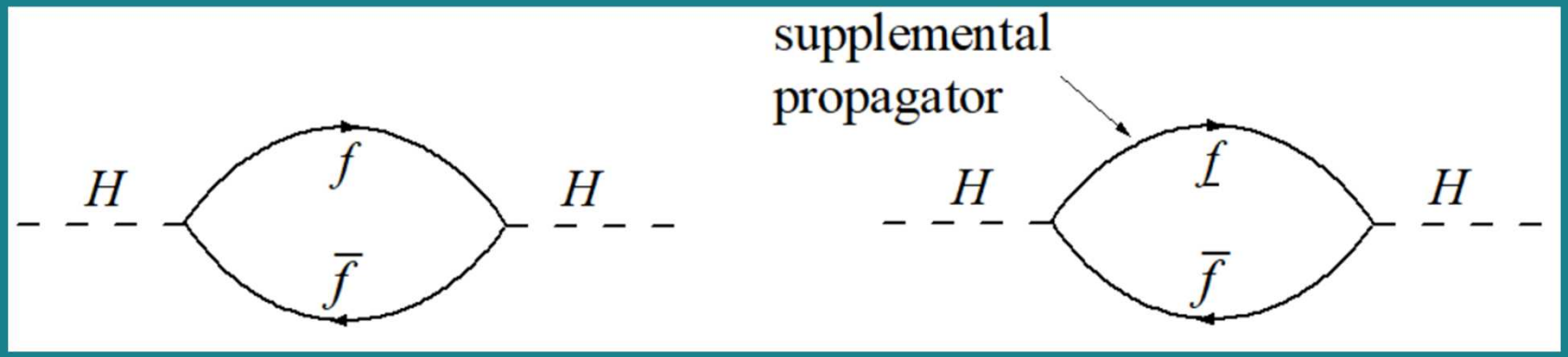
-

10 billion tons of salt

Can Supplemental Propagators Help?



Supplemental and traditional propagators have opposite signs.
Total amplitude contribution = 0.



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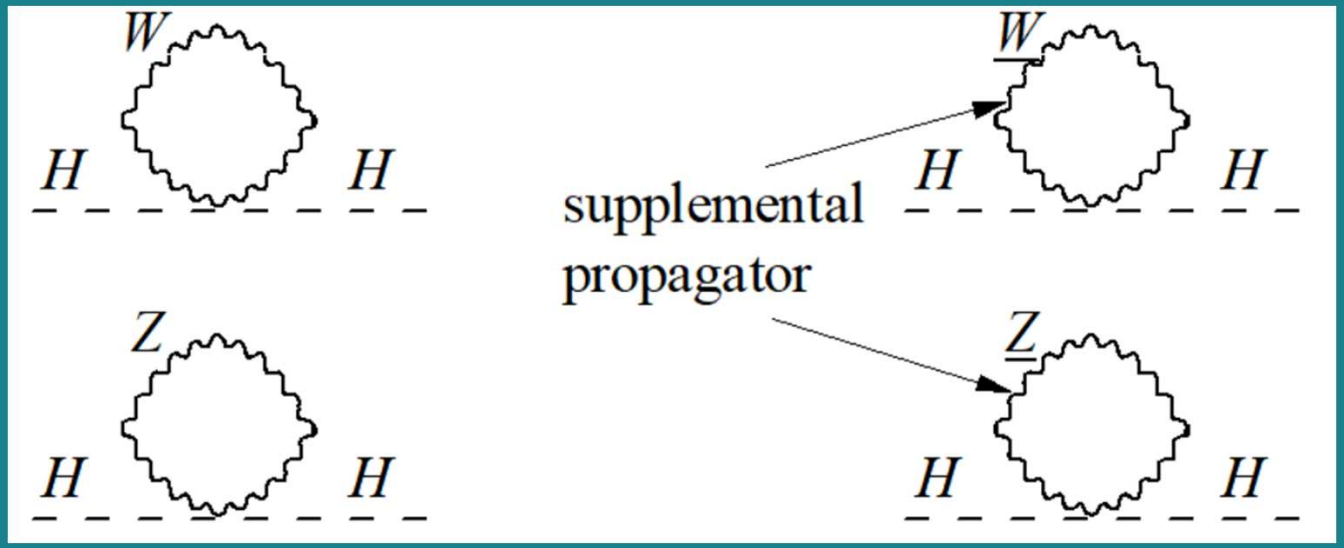

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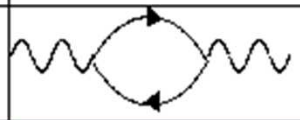
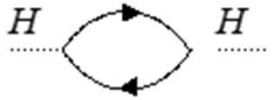
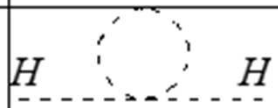
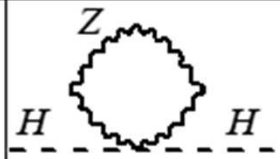
Problems and Issues

Supplemental Fields/Particles Issues

- *Fock space metric not positive definite*
  - *Half of states have negative probabilities*
  - *Appears fixable by redefining norms*
- *Deeming supplementals only virtual hard to justify with math alone*
  - *Need to simply assume supplemental particles only virtual, not real*
- *Gauge hierarchy fermion loop and W,Z loop coupling*
  - *Additional contributions to other corroborated scattering amplitudes*

A Closer Look at Gauge Hierarchy

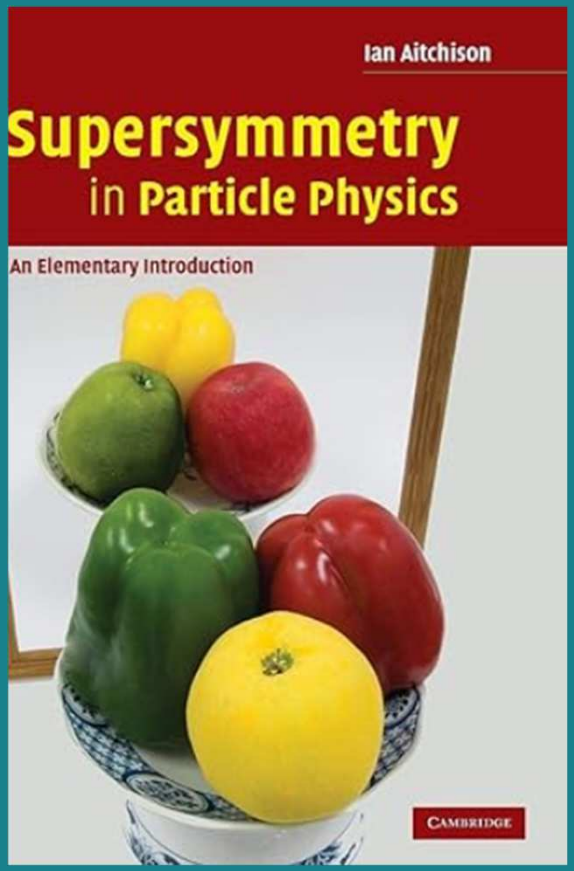
Is Something Fishy with Loop Evaluations?

Radiative Correction	Feynman Diagram	Cut-off Analysis, Naïve Divergence	Dimensional Analysis Actual Divergence
Photon internal fermion loop		Λ^2, quadratic	ln Λ, logarithmic
Higgs fermion loop		Λ^2, quadratic	Shouldn't it be ln Λ, like photon? Often stated as Λ^2
Higgs H tangent loop (λ term)		Λ^2, quadratic	Often stated as Λ^2
Higgs W tangent loop		Λ^2, quadratic	Often stated as Λ^2
Higgs Z tangent loop		Λ^2, quadratic	Often stated as Λ^2

Fermion Loop Evaluation Options

- *Cut-off method to determine fermion loop integrals*
  - *Gives wrong answer in QED : Quadratic divergence*
  - *Not Lorentz invariant*
- *Dimensional regularization for fermion loop integrals*
  - *Gives correct answer in QED: Log divergence*
  - *Lorentz invariant*

Typical Text Treatment of Gauge Hierarchy



Chapter 1

Introduction and motivation

1.1 The SM fine-tuning problem

The electroweak sector of the SM contains within it a parameter with the dimensions of energy (i.e. a 'weak scale'), namely

Fermion loop evaluation

$$\left(-g_f^2 \int^\Lambda \mathrm{d}^4k\, \mathrm{Tr}\left[\frac{1}{(\not{k}-m_f)^2}\right]\right)\phi^\dagger\phi = \left(-4g_f^4 \int^\Lambda \mathrm{d}^4k\, \frac{k^2+m_f^2}{(k^2-m_f^2)^2}\right)\phi^\dagger\phi. \qquad (1.19)$$

$$\Lambda^2\phi^\dagger\phi. \qquad (1.20)$$

Cut-off method (which doesn't work in QED)

Fermion Loops in GSW Higgs Model

- *Higgs fermion loop integral exact same as in QED*

- *But authors cite cut-off method as applicable*

  Quadratic divergence and gauge hierarchy problem

- *If used dimensional regularization*

  Log divergence and no gauge hierarchy contribution

Other Loops in GSW Higgs Model

- *H, W, Z loops not trivial to evaluate*
- *Authors cite cut-off method as applicable*
  - *Quadratic divergence and gauge hierarchy problem*
- *If used dimensional regularization*
  - *Evaluation complicated and lengthy*
  - *Might expect log divergence and no gauge hierarchy contribution*

Overview: Higgs Mass Loop Corrections

- *If use method that doesn't work in QED*

  Serious (gauge hierarchy) problem

- *If use method that works in QED:*

  No problem for fermion loops

  Likely no problem for other loops

Relevant Paper

- *Grange, Mathiot, Mutet, Werner (2013)*

Phys Rev D 88, 12, 125015

Alternative regularization method

No gauge hierarchy problem

Claim problem is artifact of using incorrect math

https://arxiv.org/abs/1312.5278

Conclusion

Bottom Line

- *Supplemental Particles*
  - *Eliminate large vacuum energy problem*
  - *May help resolve gauge hierarchy*
- *Gauge Hierarchy*
  - *May only be a problem because wrong math used*